# First Experiences with a Computer Algebra System

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The results of a survey of 120 first year mathematics students confirm earlier findings that student approaches to learning mathematics are related to their conceptions of the nature of mathematics. In addition, a positive initial experience with the computer algebra system *Mathematica* is associated with a deep approach to study, a cohesive view of mathematics, and a more extensive background in computing. Features of students' initial experiences with *Mathematica* are described, based on qualitative and quantitative data from the survey.

Research concerning the integration of computer algebra systems (CAS) into teaching and learning at the tertiary level has mainly been of two kinds, the first being descriptions and evaluations of innovative teaching programs (e.g., Houston, 1994). The second kind consists of reports of teaching experiments in which comparisons are made between groups taught in a CAS environment, and groups taught in a traditional mode with content, and in contexts, appropriate to the requirements of the latter, (e.g., Heid, 1988; Meel, 1998). These and other papers indicate that students taught in a CAS environment achieve at least as well as those taught in traditional classes on measures of pen and paper skills at the first year level, and in many cases achieve a higher level of conceptual understanding. Direct comparisons are not appropriate, of course, in cases where the use of CAS changes the type of questions that might be set as assessment tasks and how they might be approached, for example in advanced calculus subjects.

An area in which there has been only preliminary research is the set of interactions between CAS experiences and students' approaches to learning mathematics, and indeed their conceptions of the subject itself. This paper reports some early results of a survey and interview study designed to investigate the experiences of first-year mathematics students, and their lecturers, at the University of Technology, Sydney (UTS), during the incorporation into teaching and learning programs of a particular CAS, *Mathematica.*

## Background

Recent work at the University of Sydney (Crawford, Gordon, Nicholas and Prosser (1994, 1998a and b), and conducted without a CAS perspective, identified relationships between students' approaches to learning mathematics and their beliefs about the nature of mathematics. Two questionnaires were developed. One is based on the SPQ (Study Process Questionnaire) developed by John Biggs (1987). Among the items retained and revised are:

- "I think browsing around is a waste of time, so I only study seriously the mathematics that's been given our in class or in the course outline." *Surface approach subscale.*
- "I believe strongly that my aim in studying mathematics is to understand it for my own satisfaction." *Deep approach subscale.*

The Conceptions of Mathematics Questionnaire is based on an earlier phenomenographic study (Crawford et al, 1994), and has two subscales indicating a Fragmented Conception (Mathematics is seen as a collection of rules and formulas), and a Cohesive Conception (Mathematics is seen as a logical system for generating new knowledge). Examples of the items are:

- "Mathematics is a lot of rules and equations." *Fragmented subscale.*

- "Mathematics is a set of logical systems which have been developed to explain the world and relationships in it." *Cohesive subscale.*

Crawford et al (1998b) reported statistically significant correlations between these two sets of subscales, namely a positive correlation between a surface approach to study and a fragmented conception of mathematics, and a positive correlation between a deep study approach and a cohesive conception. When other variables which reflected students' perceptions of the teaching and learning environment, and prior academic ranking were included, two clusters of students were identified. These were described as follows:

> This analysis suggests that students holding cohesive conceptions of mathematics adopt deep approaches to learning mathematics, and have very different interpretations of learning mathematics at university. They perceive the learning environment as more satisfactory and fulfilling than do students reporting fragmented conceptions. Moreover, these students achieve at a higher level in their university study of mathematics than those students holding fragmented conceptions of mathematics and adopting surface approaches to learning. (Crawford et al, 1998a, p. 465)

The authors of that study emphasised that their model of student learning is not causal and deterministic, but instead indicates ongoing interrelationships between prior and post experiences and understandings, perceptions and study approaches, and the teaching and learning context.

This move away from causal models towards a descriptive framework is also seen in the development by Biggs (1991, 1993, 1999) of his 3P (Presage, Process, Product) model of student learning. The Presage components are Student (prior knowledge, abilities, preferred ways of learning...) and Teaching Context (curriculum, teaching method, classroom climate, assessment...). In this model, the introduction of a CAS is part of the Teaching Context, and will both influence, and be influenced by, the other components.

An important aspect of the model is that learning approaches are not fixed properties of an individual and are not necessarily transituational. This is where the role of *metalearning* is vital. Metalearning involves awareness of and executive control over one's learning processes (Biggs, 1987). This highlights the difference between the developed 3P model, and models based on an information processing metaphor that regard learning styles as fixed properties of the individual. Having to learn about a CAS may make a difference, then, to an individual who usually adopts a surface approach to learning mathematics, but is able to make use of CAS to carry out the personally directed investigations that usually distinguish a deep approach to mathematics.

## Method

### *The Survey*

In order to identify some of the interactions between students, the teaching context, and the kinds of task processing (approaches to study) adopted by students a survey was designed to collect information about students' conceptions of mathematics, their approaches to studying mathematics, and their experiences of learning *Mathematica.* It was expected that the relationships reported by Crawford and others for students at the University of Sydney would be identified, because of similarities in cohort, pedagogy, and context. What was not known was how students at UTS who scored differently on the various subscales of the Conceptions of Mathematics Questionnaire (CMQ) and the Study Process Questionnaire (SPQ) would vary in their reports of their CAS experience. Biographical data was also requested: age, sex, language spoken most of the time at home, prior mathematics studied, and a self-assessment of computing background.

*First Experiences of Mathematica Questionnaire (MEQ)*

The source of most of these 56 questions was the set of anonymous comments made a year earlier on subject evaluations by students responding to the open-ended question, “Overall, how would you describe your initial experiences with *Mathematica?*”. Other questions were written in relation to some of the metaphors for graphing calculators that Kissane, Kemp and Bradley (1995) identified in a study of first year mathematics students’ attitudes towards new technology. Two examples are "Mathematica was an invaluable aid to my learning of mathematical concepts", (the *Laboratory* metaphor), and "I resent the time taken on Mathematica assignments", (the *Nuisance* metaphor). Other items reflected practical experiences with the software and hardware, and opinions about the assessment. To record their responses to each item on the MEQ, students were requested to mark a position on a line that was about 4 cm long, with Disagree on the left and Agree on the right. The location of the student response on the scale was interpreted as a numerical measure of strength of agreement with each proposition. This gave a measure for each item from zero to 41 on a notionally continuous scale.

*The Students*

The target population for the survey consisted of all students enrolled in three different first year, first semester mathematics subjects. They will be referred to here as Subject 1 (for students majoring in mathematics, about 80 in this class), Subject 2 (for students in physical science courses, about 110), and Subject 3 (for students in engineering courses, about 400). The surveys were distributed in classes at the beginning of the subsequent semester but only for Subject 1 were the students allowed class time to complete the surveys: 60 were returned. The low response rates for Subjects 2 and 3 mean that comparisons across the three subjects must be made with caution, if at all. The responses as a whole do give information about the range of experiences, if not about the distribution of those experiences.

Because of choices made by the coordinating lecturers, the planned learning experiences with *Mathematica* for students in the three subjects were quite different. In Subject 1, students attended ten one-hour laboratory sessions, and were given worksheets to read and work through on the computers. These described the application of CAS to the solution of real world problems from a variety of contexts from finance to the design of theme park rides. An assignment on a different topic, models of population growth, requiring many of the commands introduced in the labs was given towards the end of semester and students spent many hours out of class working on it. In subject 2, students again attended weekly labs but had worksheets designed to introduce only those *Mathematica* commands that were relevant to the work covered in lectures that week. The worksheets contained several questions to be done with pen and paper as well as on the computer. Discussion and working in pairs was actively encouraged. The engineering students in Subject 3 had only three scheduled one-hour labs and were expected to use that time, and time out of class, to complete an assignment that reflected a main topic in the subject: mathematical modelling.

## Results

*Items from the MEQ with High and Low Means*

The means and standard deviations of the five items with which students showed most agreement are listed below in Table 1. Also shown are the five items with the lowest means, that is the items with which students disagreed most strongly.

Table 1
*Items with High and Low Scores on the MEQ*

| Item (Agreement is a high score on a scale of 0-41) | Mean | Std Dev'n. |
|---|---|---|
| 48. It was frustrating when you forgot capital letters or used the wrong brackets and it wouldn't work. | 30.4 | 10.0 |
| 44. I need more practice with *Mathematica* before I can use it as a tool. | 29.3 | 10.4 |
| 30. *Mathematica* is too expensive to buy for myself so I did not have the benefit of working on it at home. | 29.1 | 13.3 |
| 45. *Mathematica* is useful as a checking tool. | 28.1 | 9.5 |
| 11. When I get an unexpected output or error message from *Mathematica* I try to work out the reasons why before changing my input. | 28.0 | 10.3 |
| **Item (Disagreement is a low score on a scale of 0-41)** | **Mean** | **Std Dev'n.** |
| 9. I had difficulty using the printer. | 8.8 | 11.3 |
| 23. I looked for books in the library on *Mathematica.* | 8.9 | 11.8 |
| 15. I had difficulty saving my work onto floppy disks. | 9.0 | 10.4 |
| 35. I have used *Mathematica* to investigate mathematical questions of my own that arose in other subjects. | 9.7 | 10.3 |
| 17. I often used *Mathematica* to explore my own questions about mathematics. | 10.3 | 10.2 |

## *Items Reflecting the Overall Experience*

At the end of the questionnaire were two free response questions. The first was: "Overall, how would you describe your experiences with *Mathematica* last semester?" The purpose of this question was to gather information about the students' experiences in their own words, and to provide a check that issues that were important to students had indeed been reflected in the questionnaire items. The replies to the first question were transcribed and then sorted into three categories indicating a negative (47 comments), neutral (27), or positive (25) overall experience. Within each category, responses with a similar focus were grouped together and summarised. These groups are reported below with illustrative comments. Minor spelling and grammatical errors have been corrected. The numbers in each of the smaller groups are not considered important in this stage of the analysis where there is an attempt to capture the range of experiences rather than their distribution. It should be noted that many of the comments could have been placed in more than one group, and it is acknowledged that this grouping is subjective.

### Comments Revealing a Poor or Negative Overall Experience

*Time.* "Very frustrating at times. I could see how it would be very useful in helping with some maths but with the little time we had with it I wasn't able to get a hang of it very well."

*Inadequate teaching.* "I felt that they didn't really teach us how to use the program, that they expected us to be able to use the whole program after giving us only a couple of commands."

*Difficulty of Mathematica itself.* "I found it hard to understand how *Mathematica* works." "It was difficult to learn and remember all the commands."

*Assignments were too hard.* "*Mathematica* was very hard to get the hang of. Assignment was so hard that I didn't and couldn't hand anything in . I find the language very hard to understand." "The tutes were good, but the assignment was a HORROR!"

*Overall a waste of time.* "Time could be spent on more useful learning in maths or other subjects."

*Own lack of computing experience.* "Frustrating because of my lack of computing experience."

*Took too long to learn the syntax.* "Painful. A lot of effort went into learning the syntax and in the assignments this meant trying to get it to work instead of trying to get the right answer."

### Comments Revealing that the Overall Experience was Neutral or Satisfactory

*Frustrating but useful*: "It was hard to use and required a lot of work to operate. It was however useful for basic and tedious calculations in other assignments, but I had trouble applying it to complex problems."

*Frustrating at first, but a useful/worthwhile experience later after student's effort or because of the group work:* "Frustrating. But, ultimately after all the hours I put in to it, it was rewarding completing the assignment." "Neutral. I understand that *Mathematica* is a powerful tool ... I was however, unable to see the potential of

*Mathematica* – especially when applied to other subjects. This was mainly due to a lack of knowledge on commands, syntax, etc. This lack of knowledge, on the other hand, encouraged me to research on data regarding *Mathematica* and talk with my friends about maths..."

Comments Revealing that the Overall Experience was Worthwhile

*Interesting:* "A challenge, which made it interesting." "... Overall my first experience was interesting. It was good to see things that you usually can not." "It made maths a lot more interesting."

*Useful:* "I think it is a quite positive experience, because now I know there is software package that helps us to solve mathematics." "Good. I can see its use as a tool in industry."

*Rewarding:* "Big effort, but rewarding."

*Helped with maths*: "It was a new experience as before last semester I didn't even know *Mathematica* existed. Over all it tends to help one's understanding of mathematics."

In the light of this range of reported experiences, the 56 items in the MEQ were again reviewed and a shorter list of 26 items was constructed to try to capture the features of an overall positive experience with *Mathematica.* Again this was a subjective judgement although the checking against the students' written comments lends support to its being a useful choice. The average of these 26 items was then labelled *Overall Mathematica Experience.* For the whole sample the mean of this variable was 19.2 and the standard deviation was 5.7. The means for various subgroups were obtained and no statistically significant differences were found for groupings by Subject, Gender, Home Language, and Age (recent school leavers or mature age) across the whole sample. Computing Experience did make a difference, however, when the sample was divided into "low" and "high" levels of computing background.

### *Relationships with the SPQ and CMQ*

One of the aims was to investigate the relationships, if any, between scores on the subscales of the CMQ (Fragmented/Cohesive), the SPQ (Surface/Deep), and the items on the MEQ. Those which yielded statistically significant correlations greater than 0.25 are reported below in Table 2.

Table 2
*Correlations of MEQ Items with the Four Subscales of the CMQ and the SPQ*

| Item from MEQ | Fragmented Conception | Cohesive Conception | Surface Approach | Deep Approach |
|---|---|---|---|---|
| 5. In general, the Mathematica assignments were too difficult for me. | 0.28** | –0.26** | 0.22* | |
| 7. It was hard for me to understand what the Mathematica assignments were asking me to do. | 0.29** | | 0.24** | |
| 8. The Mathematica assignments were interesting. | –0.22* | 0.32** | | 0.25** |
| 11. When I get an unexpected output or error message from Mathematica I try to work out the reasons why before changing my input. | –0.30** | 0.20* | –0.19* | |
| 17. I often used Mathematica to explore my own questions about mathematics. | | 0.25** | | 0.28** |
| 18. The only time I used Mathematica was when we had to hand something in. | 0.26** | –0.25** | | |
| 22. Getting the right output from Mathematica is largely a matter of trial and error. | 0.20* | –0.36** | 0.21* | –0.25** |
| 28. I enjoyed discussing Mathematica with other students. | | 0.29** | | 0.33** |
| 29. Because of Mathematica, I had interesting conversations with others about mathematics. | | | | 0.42** |
| 31. Mathematica just added to the amount we already had to learn. | 0.28** | –0.27** | 0.20* | –0.25** |
| 47. Mathematica was fun. | | 0.37** | | 0.27** |
| 53. Creativity was adequately rewarded in the Mathematica assignments. | | 0.26** | | 0.28** |

*Note.* *Correlation is significant at the 0.05 level; **correlation is significant at the 0.01 level

The correlations between the subscales themselves are given below in Table 3, with the corresponding results from Crawford et al (1998b) shown for comparison. The correlations are in the expected directions and of comparable significance overall.

Table 3

*Correlations Between Study Process Variables and Conceptions of Mathematics Variables.*

| | Conceptions of Mathematics Variables | | | |
|---|---|---|---|---|
| | Fragmented | | Cohesive | |
| Study Process Variables | | | | |
| Surface Approach | *0.36*** | 0.20* | *–0.04* | –0.22* |
| Deep Approach | *–0.12** | –0.17 | *0.43*** | 0.58** |

*Note.* *Correlation is significant at the 0.05 level; **correlation is significant at the 0.01 level. Figures in italics from Crawford et al (1998b), N=300; figures in normal font from current study, N=120.

The correlations between Overall Mathematica Experience and the four subscales of the CMQ and the SPQ are also in the expected directions, although none of them are very large. This is shown in Table 4 below.

Table 4

*Correlations between Overall Mathematica Experience and the CMQ and SPQ Subscales*

| | Fragmented view | Cohesive view | Surface approach | Deep approach |
|---|---|---|---|---|
| Overall *Mathematica* Experience | –0.22* | 0.32** | –0.19* | 0.29** |

*Note.* *Correlation is significant at the 0.05 level; **correlation is significant at the 0.01 level.

## Discussion

There are messages in the results of this research for those designing introductory learning and assessment tasks for *Mathematica* or any CAS. A reading of Table 1 indicates that one semester's work in one hour weekly labs (or less in the case of Subject 3) was not enough time to convince most students in this sample that *Mathematica* is a productive tool for their own use. It could well be that Lecturers who have learned several "mathematical" programming languages in their careers (for example, BASIC, FORTRAN) simply underestimate the amount of time it takes to become familiar with new syntax, let alone new concepts such as assigning values to variables and defining functions. While some students were happy to teach themselves and make the necessary investments of time and reading, many expected more help from the teaching staff and from the materials provided. A possible direction for change is a complete revision of the kind of assignment (if any) that is set to be completed with *Mathematica*. Some kind of graded choice in topics might meet the need to provide challenge to students already confident in using computers, while allowing a less demanding task with more support from tutorial staff for those who need it.

In this sample, the students who have a positive first experience with *Mathematica* are not more likely to be male or female, from English or Non-English home language backgrounds, or to be older or younger. Previous computer experience does seem to help, and this echoes a finding by Galbraith, Haines, and Pemberton (1999, p. 219) that "computer attitudes are more influential than mathematical attitudes in facilitating the active engagement of computer related activities in mathematical learning."

The relationships between Conceptions of Mathematics, Approaches to Learning, and Overall *Mathematica* Experience are not strong but warrant further investigation. In this phase of the study, it is not possible of course to suggest directions for these relationships. We cannot say, for example, whether students who already had a cohesive view of mathematics found the assignments interesting because of that, or whether the nature of the assignments contributed towards that cohesive view. Crawford et al (1998a) found little change in either Approaches to learning or Conceptions of mathematics over students' first semester at university. (See Note). Whether or not a successful experience with a Computer Algebra System might encourage Surface Learners to adopt a Deeper approach, or perhaps contribute to a change in conceptions of mathematics from fragmented to cohesive, remains unanswered.

## Future Directions.

Interviews with students and staff will provide more information about the issues that have been raised. In particular, it will be useful to investigate the differences and similarities between approaches (surface/deep) to *learning mathematics,* approaches (surface/deep) to *learning Mathematica,* and approaches (surface/deep) to *learning mathematics with Mathematica.* Further refinement of the items in the MEQ would lead towards the development of a shorter questionnaire that could be used in future investigations.

## Conclusion

Among the advantages claimed for the introduction of technology (graphing calculators and computer algebra systems) into upper secondary and tertiary mathematics education are: preparing for further study and work, (Leigh-Lancaster, 1996); reducing time spent on routine calculations, thus allowing teachers and students to concentrate on concepts and the interpretation of results, (Mueller, Pedler, Anderson, & Bloom, 1998); reducing conceptual misunderstandings in mathematics by linking visual, symbolic, and numerical representations (Eisenberg & Dreyfus, 1991); stimulating a new approach to teaching mathematics through guided discovery (Leigh-Lancaster & Stevens, 1999).

Some of the students in the current study would agree with these claims, in particular those who spent time learning the syntax of the program and becoming familiar with its operating paradigm, and those who enjoyed the discussions about mathematics that arose while they worked on challenging tasks. However many are yet to realise these benefits and it is a challenge to design learning tasks, perhaps more integrated with or even replacing lectures, which will assist more students to benefit from the new technology.

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[1] *Note.* A typographical error appears in Table 3 of the reference Crawford et al 1998a. In the first column of figures, -0.25** should appear as 0.25**. (Personal communication with one of the authors of that study).

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